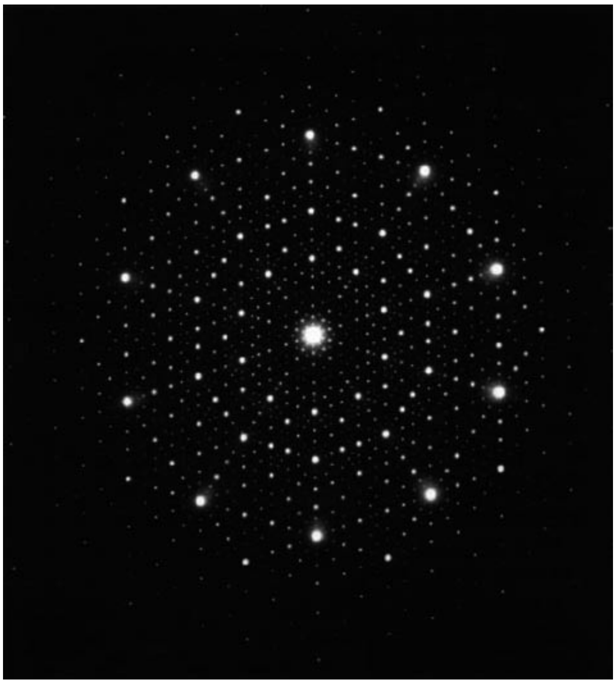
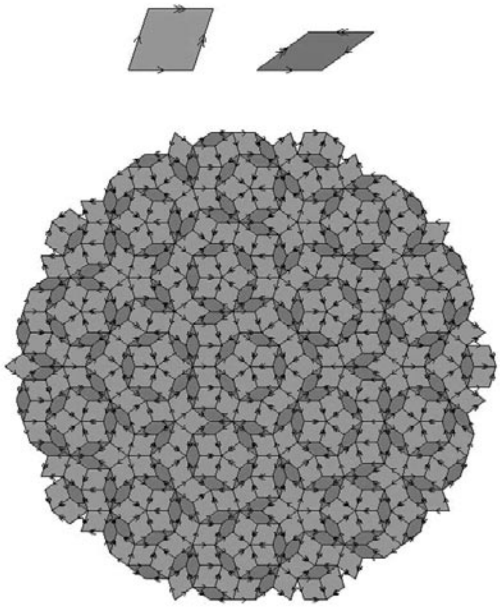
On Quasicrystals

The greatness of the discovery of quasicrystals by Dan Shechtman is not that they have so many wonderful new uses – in fact they have had relatively few exciting applications – but because they represent a brand new idea: a form of order in Nature that no one suspected to exist. In fact the substantial belief was that such materials could not exist. Shechtman discovered that there are extended atomic structures that are almost periodic (with this having a precise meaning) that permit symmetries to appear that cannot exist in crystals. The most important manifestation of quasicrystals is their implicit long-range internal order that makes itself apparent in the beautiful and perfect diffraction patterns associated with them. By now over 150 quasicrystal materials are known, some of them showing perfection that rivals the best crystals.



Selected Area Electron Diffraction pattern of a decagonal phase of the $Al^{70}Co^{11}Ni^{19}$ quasicrystal alloy.[1]

The discovery of these materials immediately engaged the great interest of theorists – physicists and mathematicians. In fact with the discovery of physical quasicrystals came the realization that mathematical models of this type of almost periodic order had already appeared in mathematics earlier, in the form of Penrose and other aperiodic tilings and in the theory of model sets created by Yves Meyer while studying Diophantine approximation.



Penrose tiling

With Shechtman's discovery came the exciting news that Nature knew about them too. By now, over 25 years later, a significant new area of almost periodic order has sprung up. This area is highly interdisciplinary, incorporating ideas from statistical mechanics, diffraction, the Fourier analysis of infinite measures, algebraic number theory, discrete geometry, the study of self-similar structures and fractal measures, the cohomology of C*-algebras, dynamical systems, and the study of aperiodic Schroedinger operators.

Almost periodic order appears wherever there are incommensurate magnitudes that are competing with one another, and because of this it can occur at any scale, not just at the atomic scale. Probably the best way to think of the subject is as a probe into the vast and largely unknown world of structures that lie between the perfectly ordered (crystals) and the completely disordered (gases). Between these two extremes lies practically everything of interest in this universe.

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Professor Moody wrote this brief article for "PAUza Akademicka". (AMK)

[1] From: S. Ritsch, O. Radulescu, C. Beeli, D.H. Warrington, R. Lück and K. Hiraga, *A stable one-dimensional quasicrystal related to decagonal Al-Co-Ni*. Philosophical Magazine Letters 80, 107–118 (2000).

[2] http://www.math.ualberta.ca/~rvmoody/rvm/